Nuclear Spin :- Neucleons (Proton and neutrons) have intrinsic spin angular momentum $\frac{1}{2}$ (in unit of $\hbar$) each like electrons. Neucleons also have ~~an~~ quantized orbital angular momentum about the centre of mass of the nucleus. The resultant angular momentum I of the nucleus is the vector sum of the orbital angular momentum L and Spin angular momentum S of the nucleus.

$$I = L + S$$

Quantum mechanically they are represented as $P_I^2 = I(I+1)\hbar^2$, $P_L^2 = L(L+1)\hbar^2$, $P_S^2 = S(S+1)\hbar^2$

In the time of measurement it is the largest component of the angular momentum along the direction of applied electric or magnetic field which is determined.

Now the total spin angular momentum of nucleus is $S = \sum S_i$ where S_i is the spin angular momentum of each nucleon. Same as total orbital angular momentum can be represented $L = \sum l_i$. Since $S_i = \pm\frac{1}{2}\hbar$, S can be zero/ integral or half integral depending upon A if it is even or odd. On the other hand l_i is integral (0, 1, 2..) so L can be integral or zero.

So, the total angular momentum I of nucleus can be either integral for even A or half odd integral for odd A which is ~~obsert~~ observed in experiment.

The total angular momentum of nucleus I is currently referred to as nuclear "Spin" even though 'Spin' term is reserved usually for intrinsic angular momentum of elementary particles.

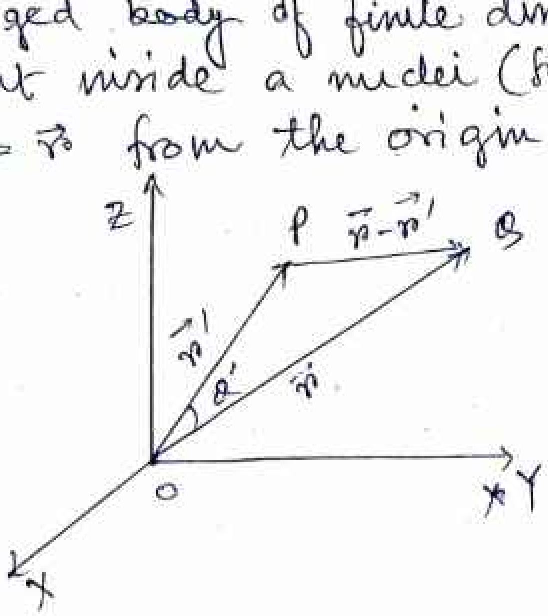
It has been found that ground state spin of nuclei for even Z even N nucleus, is invaribly zero (I=0). This result indicates the tendency of the nucleons inside the nucleus to form pairs with equal and opposite, aligned angular momentum, which cancel out in pairs for like nucleons.

The nuclear spins are found relatively small, largest measured value of ground state spin is 9/2 and ~~for~~ $I \geq 7$ for one nucleus ^{176}Lu. This value are much less than the value $\frac{A}{2}$ which could ~~b~~ be expected if all intrinsic spins are parallel. This is conformity regarding pair formation within nucleus. Majority of the nucleons of either type seems to form core in which even no of protons and nutrons are grouped in pair so that total angular momentum of core becomes zero. So, only few nucleons remaining outside the core determine the nuclear spin which is thus small integral or half odd integeral.

Electric moments of nuclei

We have considered previously that nucleus is spherically symmetric charged body. Actually most of the nucleus are spheroidal and ~~it has~~ they have rotating cylindrical symmetric axis.

The atomic ~~nou~~ nucleus is a positively charged body of finite dimension. Let P is a point inside a nuclei (fig 1) at a distance $OP = \vec{r}$ from the origin O.



If charge density at P is $\rho(\vec{r}')$ [ρ is not uniform] and small volume sorrounding the point P is $d\tau'$, then charge ~~will be~~ at that point is $\rho(r')\,d\tau'$.

So the potential at point Q, due to charge at point P is

$$dV(r) = \frac{1}{4\pi\epsilon_0} \frac{\rho(r')\,d\tau'}{|\vec{r}-\vec{r}'|} \quad \text{.....} (1)$$

$$\therefore |\vec{r}-\vec{r}'|^{-1} = (r^2 - 2rr'\cos\theta' + r'^2)^{-1/2} \quad [\theta' \text{ is angle between } r \text{ and } r']$$

$$= \frac{1}{r}\left[1 - \left(\frac{2r'}{r}\right)\cos\theta' + \left(\frac{r'}{r}\right)^2\right]^{-1/2}$$

$$= \frac{1}{r}\left[1 + \frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2 \frac{1}{2}(3\cos^2\theta' - 1) + \cdots\right] \quad \text{.....} (2)$$

It can be represented with Legendre polynomial also

$$dV(r) = \frac{\rho(r')}{4\pi\epsilon_0 r}\left[\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\theta')\right]d\tau \quad \text{.....} (3)$$

So, we can find the total potential at point Q due to the total nucleus using eqns (1) and (2)

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \rho(r')\frac{1}{r}\left[1 + \frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2 \frac{1}{2}(3\cos^2\theta' - 1) + \cdots\right]d\tau \quad \text{....} (4)$$

Let, Q point is situated at Z axis and $r'\cos\theta' = z'$. So, θ' is the angle between radial vetor r' and axis of rotation or symmetric axis Z. Then the above eqn can be written as

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{e}{r}\left[Z + \frac{D}{r} + \frac{1}{2}\frac{Q}{r^2} + \cdots\right] \quad \text{....} (5)$$

Here

$$Z = \frac{1}{e}\int \rho(r')\,d\tau' \quad \text{......} (6)$$

$$D = \frac{1}{e}\int \rho(r')\,z'\,d\tau' \quad \text{......} (7)$$

$$Q = \frac{1}{e}\int \rho(r')(3z'^2 - r'^2)\,d\tau' \quad \text{....} (8)$$

Z is atomic no of nucleus. D and Q are Dipole moment and Quadrupole moment of nucleus. Dimension of D and Q are Length and Area.

Now quantum mechanically $\rho(r')/e$ should be replaced by $|\Psi|^2$ in the above eqns.

It is an even function. As D is an odd function so the total integration of D over total nucleus must be zero. Therefore, there is no exist of dipole moment of nucleus. Similarly, ~~Octum~~ octupole moment can not exist. But quadrupole moment as it is even function, can exist.

Geometrical significance of electric quadrupole moment of nuclei:

Let, ~~nucleu~~ nuclei is spheroidal (ellipsoidal) charge distribution with uniform charge density. Suppose, there is two co-ordinate system at the centre of the nuclei. One is body fixed co-ordinate system (x', y', z') and other is space fixed co-ordinate system (x, y, z). Let, z' axis is the symmetric axis of nuclei and β is angle between z and z' axes which are shown in fig 2.

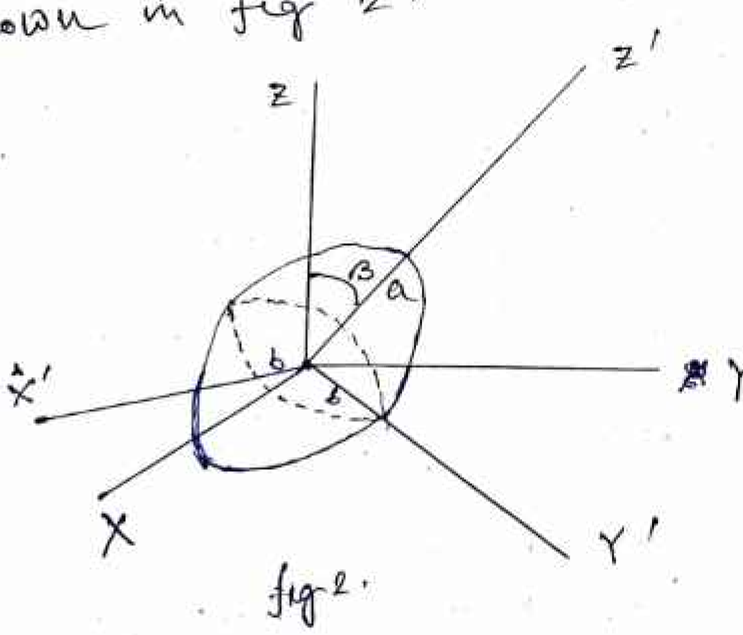


fig 2.

Ellipsoidal eqn in body fixed co-ordinate system is
$$\frac{x'^2}{b^2} + \frac{y'^2}{b^2} + \frac{z'^2}{a^2} = 1$$

Assuming uniform charge distribution with density ρ, we have for a nucleus of charge Ze.

$$\rho = \frac{Ze}{\frac{4}{3}\pi a b^2} \quad \ldots \quad (10)$$

As z' axis is the symmetric axis of nucleus then it has cylindrical symmetry about that axis. In cylindrical polar co-ordinates (s', ϕ', z') the eqn (9) reduces to:

$$\frac{s'^2}{b^2} + \frac{z'^2}{a^2} = 1 \qquad \text{where } s'^2 = x^2 + y^2 \quad \ldots \quad (11)$$

$$\text{or} \quad s'^2 = b^2\left(1 - \frac{z'^2}{a^2}\right)$$

Then we get from the equation (8)

$$Q_B = \frac{1}{e}\iiint \rho\left(3z'^2 - r'^2\right) d\tau'$$

$$\text{or,} \quad Q_B = \frac{3Z}{4\pi a b^2}\int_{-a}^{+a} dz' \int_0^{2\pi} d\phi' \int_0^{b\left(1-\frac{z'^2}{a^2}\right)^{1/2}} \left(2z'^2 - s'^2\right) s'\, ds'$$

$$\text{or} \quad Q_B = \frac{2}{5} Z \left(a^2 - b^2\right) \quad \ldots \quad (12)$$

In equation (12),

(i) <u>if $a = b$</u> then the quadrupole moment of the nucleus will be zero. So, spherical nuclei have no quadrupole moment ($Q = 0$). [fig. 3

ii) <u>if $b > a$</u> In this case $Q < 0$. So, the quadrupole moment of oblate shaped nuclei must be negative. [fig 4]

iii) <u>if $a > b$</u> In this case $Q > 0$. So, the quadrupole moment of prolate shaped nuclei must be positive. [fig 5]

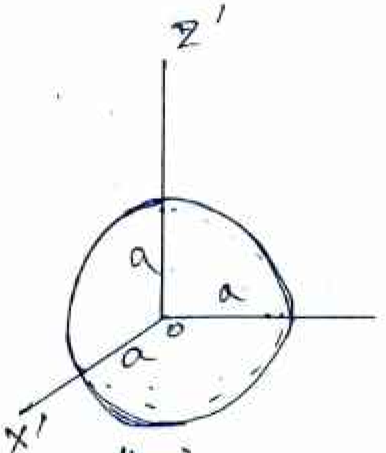


fig 3

i) Spherical nuclei ($Q_0 = 0$)

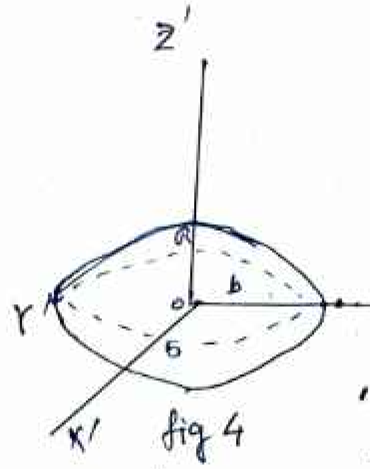


fig 4

(ii) Oblate nuclei ($Q_B < 0$)

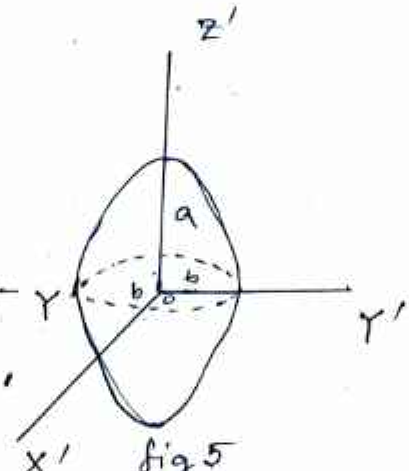


fig 5

(iii) Prolate nuclei ($Q_0 > 0$)

Writting the eccentricity parameter $\epsilon = \dfrac{a^2 - b^2}{a^2 + b^2}$, and mean square radius of the nucleus $\langle R^2 \rangle = \dfrac{a^2 + b^2}{2}$, we then get from eqn. (12)

$$Q_B = \frac{4}{5} Z\epsilon \langle R^2 \rangle \quad \text{---- (13)}$$

Thus substituting the value of $\langle R^2 \rangle$ and Q_B we can find eccentricity parameter ϵ which will tell us that how much nucleus is deformed. So, the shape of nuclei depends upon quadrupole moment. Measurement gives for the deutron $Q_B = + 2.82$ milibarns $= + 2.82 \times 10^{-31}\ m^2$ which shows that charge distribution in the ^2H nucleus has the shape of prolate spheroid.

Now, we have to find the expression for quadrupole moment of nucleus according to space fixed co-ordinate, Q_s which is experimentally observed value.

We get from eqm (8) for arbitarily taken cordinate (r, θ, ϕ),

$$Q_s = \frac{1}{e} \int r^2 (3\cos^2\theta - 1) \rho \, d\tau \quad (14)$$

where $\cos\theta = \cos\theta' \cos\beta + \sin\theta' \sin\beta \cos(\phi' - \phi)$

previously we know, β is angle between Z and Z' axes.

$\therefore \quad Q_s = \frac{1}{2}\left(3\cos^2\beta - 1\right) Q_B \quad \cdots\cdots \quad (15)$

If I is the total angular momentum of nucleus then we know that $\cos\beta = \frac{I_z}{[I(I+1)]^{1/2}}$

$\cos\beta$ will be maximum when $I_z = I$, then

$$(Q_s)_{max} = (Q_s)_{I_z = I}$$

$$= \frac{2I-1}{2(I+1)} Q_B \quad \cdots\cdots \quad (16)$$

Using (14), (15) and (16), we get for any spin component I_z, $Q_{I_z} = \frac{3I_z^2 - I(I+1)}{I(2I-1)} (Q_s)_{max} \quad \cdots \quad (17)$

~~Thus, we get $Q_s = 0$ from (16) when $I = \frac{1}{2}$ or $I = 0$ (even-even nuclei) even if $Q_B \neq 0$. For $I > 1$, Q_s is non zero if $Q_B \neq 0$. For $I = 1$ $Q_s/Q_B = 0.1$ ... for $I \gg 1$ $\frac{Q_s}{Q_B} \to 1$.~~

The physical interpretation of eqn (16) is that only a fraction of the intrinsic quadrupole moment Q_B can interact with the external field.

We get $Q_s = 0$ from eqn (16) when $I = \frac{1}{2}$ or $I = 0$ even if $Q_B \neq 0$. for $I > 1$ Q_s is non zero if $Q_B \neq 0$ and for $I \gg 1$ Q_s approaches to Q_B. Vanishing of quadrupole moment (Q_s) for $I = \frac{1}{2}$ and $I = 0$ does not mean that nucleus is spherical shape but rather that it does not provide a measure of its deviation from spherical charge distribution.

Quadrupole moment for numerous nuclei has been measured using optical hyperfine structure and atomic beam technique, their values are found to range from -1×10^{-28} to $+8 \times 10^{-28}$ m^2, the values being particularly high for rare earth elements.